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Motion of a spherical particle in a rarefied gas. Part 2. Drag and thermal polarization

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Kinetic theory for the drag and thermal polarization of a spherical particle in a low-speed flow of a rarefied gas is presented. The problem is solved on the basis of the linearized kinetic equation (Shakhov 1974) with the correct Prandtl number, $Pr = \frac{2}{3}$, for monatomic gas. The integral-moment method of solution for arbitrary values of the Knudsen number is employed. The possibility of arbitrary energy, and tangential and normal momentum accommodation of gas molecules on the particle surface is taken into account in the boundary condition. The particle–gas heat conductivity ratio Λ is assumed to be arbitrary.

Numerical results for the isothermal drag, radiometric force affecting a non-uniformly heated particle in a rarefied gas, and temperature drop between the ends of the particle diameter owing to its thermal polarization in a gas flow have been obtained. The analytical expressions approximating the numerical calculations for the whole range of Knudsen numbers are given. The results obtained are compared to the available theoretical and experimental data.

1. Introduction

The problem of a low-speed rarefied gas flow past a spherical particle has received considerable attention in the general investigations of aerosol mechanics (Brock 1980). However, the gas and particle temperatures were traditionally assumed to be equal, since the particle heating in the retarding gas flow is proportional to the square of the Mach number (Landau & Lifshitz 1966) and for a subsonic flow it is negligible. In particular Cercignani & Pagani (1968), Cercignani, Pagani & Bassanini (1968) and Lea & Loyalka (1982) calculated the isothermal drag at small values of the Mach number (M) and arbitrary Knudsen numbers (Kn) on the basis of the linearized Bhatnagar, Gross & Krook (1954) kinetic equation with the boundary condition of diffuse molecular scattering by the particle surface.

Derjaguin & Bakanov (1962) have theoretically predicted the effect of particle 'thermal polarization' in a low-speed rarefied gas flow, consisting of the non-uniform temperature distribution on the particle surface. Its mean temperature there remains equal to the free-stream gas flow temperature. This linearity in M was found to be determined by the isothermal heat transfer in a moving gas (Prigogine 1955) and unrelated to the energy dissipation in the retarding gas flow near the particle. Later, a particle thermal-polarization theory at $Kn \ll 1$, based on solving the Stokes–Fourier equations taking into account the Knudsen layer near the particle surface and employing the methods of irreversible thermodynamics, was developed by Bakanov, Derjaguin & Roldughin (1979), Bakanov *et al.* (1983) and Roldughin (1987). Sone & Aoki (1977) and Vestner & Waldmann (1977) have also taken into account the particle thermal-polarization effect in calculating the drag at $Kn \ll 1$.

Experimental investigations of the particle thermal polarization were conducted by Bakanov & Vysotsky (1980), Bakanov *et al.* (1983) and Bakanov, Vysotsky & Nekrasov (1986).

Besides the temperature non-uniformity of the particle surface, the temperature non-uniformity of the surrounding gas must also be taken into account. This effect may by analogy be defined as the 'surrounding gas thermal polarization'. It was considered by Law & Loyalka (1986) at arbitrary Knudsen numbers and by Aoki & Sone (1987) at $Kn \ll 1$ and $Kn \rightarrow \infty$. The problem was solved on the basis of the model kinetic equation of Bhatnagar *et al.* (1954) but not using the correct Prandtl number for monatomic gases, and ignoring the particle thermal-polarization effect, which is only valid for cases of complete particle heat conductivity.

The purpose of this work is the elaboration of a consistent kinetic theory for the drag of a spherical particle with non-uniform surface temperature and the particle thermal-polarization effect at arbitrary Kn. The problem is solved on the basis of an approximate kinetic equation (Shakhov 1974), allowing the correct description of the combined processes of heat and momentum transfer in gases, which is the necessary condition for the problem solution. The analysis of the particle thermal-polarization value and also the isothermal and radiometric components of the dependence of the drag on the accommodation and thermophysical properties of the particle at various Knudsen numbers seem also to be of considerable interest.

2. The problem

Consider a monatomic rarefied gas flow past a spherical particle of radius R_0 at $M \ll 1$. The undisturbed values of the gas flow velocity, number density and temperature far from the particle are $\boldsymbol{U}_\infty$, n_∞ and T_∞ respectively. If the origin of the coordinates is assumed to be at the particle centre, then with the approximation of linearity in M the molecules' velocity distribution function can be written as

$$f(\boldsymbol{x}, \boldsymbol{v}) = f_\infty[1 + 2\boldsymbol{c}\cdot\boldsymbol{u}_\infty + h(\boldsymbol{x}, \boldsymbol{v})], \tag{1}$$

where $f_\infty = n_\infty \left(\dfrac{m}{2\pi k T_\infty}\right)^{\frac{3}{2}} \exp(-c^2)$, $\boldsymbol{c} = \boldsymbol{v}\left(\dfrac{m}{2kT_\infty}\right)^{\frac{1}{2}}$, $\boldsymbol{u}_\infty = \boldsymbol{U}_\infty\left(\dfrac{m}{2kT_\infty}\right)^{\frac{1}{2}}$,

$\boldsymbol{x}$ is the radius vector from the particle centre, $\boldsymbol{v}$ is the molecular velocity vector, h is the distribution function disturbance due to the particle presence.

Let the particle surface temperature be slightly different from the free-stream gas flow temperature T_∞, i.e.

$$T_s(\theta_0) = T_\infty[1 + \tau_s(\theta_0)], \quad |\tau_s| \ll 1, \tag{2}$$

where θ_0 is the polar angle on the particle surface (the problem's geometry is given in figure 1 of Beresnev, Chernyak & Suetin 1987). Let us consider the effect of the surface temperature non-uniformity on the drag without discussing as yet the physical causes for the difference between the temperatures T_s and T_∞.

The distribution function disturbance in (1), h, satisfies the linearized kinetic equation (Shakhov 1974) which may be written here as

$$\boldsymbol{v}\frac{\partial h}{\partial \boldsymbol{x}} = \frac{P_\infty}{\eta}[\nu + 2\boldsymbol{c}\cdot\boldsymbol{W} + (c^2 - \tfrac{3}{2}\tau) + \tfrac{4}{15}(c^2 - \tfrac{5}{2})\,\boldsymbol{c}\cdot\boldsymbol{S} - h], \tag{3}$$

where $P_\infty = n_\infty kT_\infty$, η is the gas viscosity at temperature T_∞.

The dimensionless quantities ν, $\boldsymbol{W}$, τ and $\boldsymbol{S}$ characterize respectively the gas density, its macroscopic velocity and temperature disturbances and the dimensionless heat flow:

$$\begin{bmatrix} \nu \\ \boldsymbol{W} \\ \tau \\ \boldsymbol{S} \end{bmatrix} = \begin{bmatrix} \dfrac{n-n_{\infty}}{n_{\infty}} \\ \boldsymbol{u}-\boldsymbol{u}_{\infty} \\ \dfrac{T-T_{\infty}}{T_{\infty}} \\ \dfrac{\boldsymbol{q}}{P_{\infty}\left(\dfrac{2kT_{\infty}}{m}\right)^{\frac{1}{2}}} \end{bmatrix} = \pi^{-\frac{3}{2}} \int \begin{bmatrix} 1 \\ \boldsymbol{c} \\ \frac{2}{3}(c^2-\frac{3}{2}) \\ \boldsymbol{c}(c^2-\frac{5}{2}) \end{bmatrix} h \exp(-c^2)\, \mathrm{d}\boldsymbol{c}, \tag{4}$$

where $\boldsymbol{q}$ is the heat flow in the gas due to the particle surface, T_s, and gas, T_{∞}, temperature difference (non-isothermal heat transfer) and also due to the gas motion (isothermal heat transfer).

The solution (3) requires the introduction of a boundary condition taking into account discontinuity in the character of the molecular velocity space of the distribution function. For the molecules incident on the particle surface we admit the possibility of the arbitrary normal and tangential momentum and energy accommodation. To take this possibility into account the reflected molecules distribution function disturbance (1) is approximated by an expansion in terms of Hermite polynomials in the velocity half-space $(\boldsymbol{n}\cdot\boldsymbol{v}) > 0$ (Shen 1967):

$$h^{+} = a_0 + a_{1n} c_r + a_{1\tau} c_{\theta} + a_2(c^2-\tfrac{3}{2}) + \dots . \tag{5}$$

The unknown expansion coefficients a_i, which depend on the polar angle θ_0, are derived in terms of Knudsen accommodation coefficients from the laws of conservation of total flow number N, tangential ($P_{r\theta}$) and normal (P_{rr}) momentum; and energy (E_r) radial flows on the particle surface (Kogan 1969):

$$\left.\begin{aligned} |N^{+}| &= |N^{-}|, \\ |P^{+}_{r\theta}| &= (1-\alpha_{\tau})\,|P^{-}_{r\theta}|, \\ |P^{+}_{rr}| &= \alpha_n |P^{s}_{rr}| + (1-\alpha_n)\,|P^{-}_{rr}|, \\ |E^{+}_{r}| &= \alpha_E |E^{s}_{r}| + (1-\alpha_E)\,|E^{-}_{r}|. \end{aligned}\right\} \tag{6}$$

Here the superscripts + and − respectively refer to quantities determined by the reflected and incident molecules; the s superscript refers to flows in the case of complete accommodation on the surface.

The force affecting the particle is determined from

$$\boldsymbol{F} = \boldsymbol{n}_z m \int_S \mathrm{d}S \int_v V_r V_z f(R_0, \boldsymbol{v})\, \mathrm{d}\boldsymbol{v}, \quad \boldsymbol{V} = \boldsymbol{v} - \boldsymbol{U}_{\infty}, \tag{7}$$

where $\boldsymbol{n}_z$ is the unit vector along the direction $\boldsymbol{U}_{\infty}$; integration is carried out over the whole sphere surface S. Equation (7) is equivalent to the drag definition usually accepted in fluid mechanics (Batchelor 1967).

3. Integral moment equations

Kinetic equation (3) with the boundary condition (5) is formally integrated along the characteristics (Marchuk 1961) and according to (4) is transformed into a set of integral equations for the macroparameter ν, $\boldsymbol{W}$, τ and $\boldsymbol{S}$ disturbances:

$$\left.\begin{aligned}
\nu &= \pi^{-\frac{3}{2}}\nu^0+\pi^{-\frac{3}{2}}\int_V [\nu J_1+2(\boldsymbol{W}\cdot\boldsymbol{\Omega})J_2+\tau(J_3-\tfrac{3}{2}J_1)\\
&\quad +\tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega})(J_4-\tfrac{5}{2}J_2)]\frac{\mathrm{d}\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^2},\\
\boldsymbol{W} &= \pi^{-\frac{3}{2}}\boldsymbol{W}^0+\pi^{-\frac{3}{2}}\int_V [\nu J_2+2(\boldsymbol{W}\cdot\boldsymbol{\Omega})J_3+\tau(J_4-\tfrac{3}{2}J_2)\\
&\quad +\tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega})(J_5-\tfrac{5}{2}J_3)]\frac{\mathrm{d}\boldsymbol{r}'\boldsymbol{\Omega}}{|\boldsymbol{r}-\boldsymbol{r}'|^2},\\
\tau &= \tfrac{2}{3}\pi^{-\frac{3}{2}}\tau^0+\tfrac{2}{3}\pi^{-\frac{3}{2}}\int_V [\nu J_3-\tfrac{3}{2}J_1)+2(\boldsymbol{W}\cdot\boldsymbol{\Omega})(J_4-\tfrac{3}{2}J_2)\\
&\quad +\tau(J_5-3J_3+\tfrac{9}{4}J_1)+\tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega})(J_6-4J_4+\tfrac{15}{4}J_2)]\frac{\mathrm{d}\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^2},\\
\boldsymbol{S} &= \pi^{-\frac{3}{2}}\boldsymbol{S}^0+\pi^{-\frac{3}{2}}\int_V [\nu(J_4-\tfrac{5}{2}J_2)+2(\boldsymbol{W}\cdot\boldsymbol{\Omega})(J_5-\tfrac{5}{2}J_3)\\
&\quad +\tau(J_6-4J_4+\tfrac{15}{4}J_2)+\tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega})(J_7-5J_5+\tfrac{25}{4}J_3)]\frac{\mathrm{d}\boldsymbol{r}'\boldsymbol{\Omega}}{|\boldsymbol{r}-\boldsymbol{r}'|^2}.
\end{aligned}\right\} \quad (8)$$

Here
$$J_n(t)=\int_0^\infty c^n\exp\left(-c^2-\frac{t}{c}\right)\mathrm{d}c$$

is the Abramowitz (1953) function; $\boldsymbol{\Omega}=\boldsymbol{r}-\boldsymbol{r}'/|\boldsymbol{r}-\boldsymbol{r}'|=\boldsymbol{c}/|\boldsymbol{c}|$; the integration in (8) is carried out over the volume V, the points of which can be reached from point $\boldsymbol{r}$ without intersecting the sphere; and the argument of the integrals J_n is $|\boldsymbol{r}-\boldsymbol{r}'|$. The dimensionless radius vector $\boldsymbol{r}$ from the centre of the particle is determined from the radius vector $\boldsymbol{x}$ as

$$\boldsymbol{r}=\frac{P_\infty}{\eta}\left(\frac{m}{2kT_\infty}\right)^{\frac{1}{2}}\boldsymbol{x};$$

and the dimensionless particle radius $R=|\boldsymbol{r}_0|=\frac{1}{2}\pi^{\frac{1}{2}}Kn^{-1}$, where the Knudsen number $Kn=l/R_0$. The mean free path of the molecules l is related to the gas viscosity coefficient η by

$$\eta=\tfrac{1}{2}mn\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}l.$$

The expressions for the absolute terms in (8) have the following form:

$$\left.\begin{aligned}
\nu^0 &= \int_{\omega_0} [a_0 J_2 + A J_3 + a_2(J_4 - \tfrac{3}{2}J_2)]\,\mathrm{d}\omega,\\
\boldsymbol{W}^0 &= \int_{\omega_0} [a_0 J_3 + A J_4 + a_2(J_5 - \tfrac{3}{2}J_3)]\,\boldsymbol{\Omega}_0\,\mathrm{d}\omega,\\
\tau^0 &= \int_{\omega_0} [a_0(J_4 - \tfrac{3}{2}J_2) + A(J_5 - \tfrac{3}{2}J_3) + a_2(J_6 - 3J_4 + \tfrac{9}{4}J_2)]\,\mathrm{d}\omega,\\
\boldsymbol{S}^0 &= \int_{\omega_0} [a_0(J_5 - \tfrac{5}{2}J_3) + A(J_6 - \tfrac{5}{2}J_4) + a_2(J_7 - 4J_5 + \tfrac{15}{4}J_3)]\,\boldsymbol{\Omega}_0\,\mathrm{d}\omega,
\end{aligned}\right\} \tag{9}$$

where $\boldsymbol{\Omega}_0 = \boldsymbol{r} - \boldsymbol{r}_0/|\boldsymbol{r} - \boldsymbol{r}|$; $\boldsymbol{r}_0$ is the radius vector on the sphere surface;

$$A = A_{1n}(\boldsymbol{\Omega}_0\cdot\boldsymbol{n}) + a_{1\tau}(\boldsymbol{\Omega}_0\cdot\boldsymbol{\tau}) - 2(\boldsymbol{\Omega}_0\cdot\boldsymbol{u}_\infty);$$

$\boldsymbol{n}$ and $\boldsymbol{\tau}$ are the normal and tangent unit vectors to the sphere surface at the point $\boldsymbol{r}_0$; ω_0 is the solid angle under which the sphere is seen from the point $\boldsymbol{r}$; and the argument of the J_n is $|\boldsymbol{r} - \boldsymbol{r}_0|$.

The set of integral equations (8), (9) containing the unknown quantities a_0, a_{1n}, $a_{1\tau}$, a_2 is completed by (6) transformed into the following integral form:

$$\left.\begin{aligned}
&\tfrac{1}{2}\pi^{-\frac{1}{2}}[a_0 + \tfrac{1}{2}\pi^{\frac{1}{2}}a_{1n} + \tfrac{1}{2}a_2 + \pi^{\frac{1}{2}}u_\infty\cos\theta_0]\\
&\quad = \pi^{-\frac{3}{2}}\int_V [\nu J_2 - 2(\boldsymbol{W}\cdot\boldsymbol{\Omega}_0)J_3 + \tau(J_4 - \tfrac{3}{2}J_2) - \tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega}_0)(J_5 - \tfrac{5}{2}J_3)]\frac{\mathrm{d}\boldsymbol{r}\,\Omega_{0n}}{|\boldsymbol{r} - \boldsymbol{r}_0|^2},\\
&\tfrac{1}{2}\pi^{-\frac{1}{2}}[\tfrac{1}{2}a_{1\tau} + (1 - \alpha_\tau)u_\infty\sin\theta_0]\\
&\quad = -(1 - \alpha_\tau)\pi^{-\frac{3}{2}}\int_V [\nu J_3 - 2(\boldsymbol{W}\cdot\boldsymbol{\Omega}_0)J_4 + \tau(J_5 - \tfrac{3}{2}J_3) - \tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega}_0)(J_6 - \tfrac{5}{2}J_4)]\frac{\mathrm{d}\boldsymbol{r}\,\Omega_{0n}\,\Omega_{0\tau}}{|\boldsymbol{r} - \boldsymbol{r}_0|^2}\\
&\tfrac{1}{2}\pi^{-\frac{1}{2}}[a_{1n} + \tfrac{1}{2}\pi^{\frac{1}{2}}(1 - \alpha_n)(a_0 + a_2 + 4\pi^{-\frac{1}{2}}u_\infty\cos\theta_0)]\\
&\quad = \pi^{-\frac{3}{2}}(1 - \alpha_n)\int_V [\nu J_3 - 2(\boldsymbol{W}\cdot\boldsymbol{\Omega}_0)J_4 + \tau(J_5 - \tfrac{3}{2}J_3) - \tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega}_0)(J_6 - \tfrac{5}{2}J_4)]\frac{\mathrm{d}\boldsymbol{r}\,\Omega_{0n}^2}{|\boldsymbol{r} - \boldsymbol{r}_0|^2},\\
&\pi^{-\frac{1}{2}}\{(1 - \alpha_E)[a_0 + \tfrac{1}{2}a_2 + \tfrac{5}{8}\pi^{\frac{1}{2}}(a_{1n} + 2u_\infty\cos\theta_0)] + a_2 - \alpha_E\tau_s\}\\
&\quad = \pi^{-\frac{3}{2}}(1 - \alpha_E)\int_V [\nu J_4 - 2(\boldsymbol{W}\cdot\boldsymbol{\Omega}_0)J_5 + \tau(J_6 - \tfrac{3}{2}J_4) - \tfrac{4}{15}(\boldsymbol{S}\cdot\boldsymbol{\Omega}_0)(J_7 - \tfrac{5}{2}J_5)]\frac{\mathrm{d}\boldsymbol{r}\,\Omega_{0n}}{|\boldsymbol{r} - \boldsymbol{r}_0|^2}.
\end{aligned}\right\} \tag{10}$$

The integration in (10) is carried out over half-space V above the plane tangent to the sphere at point $\boldsymbol{r}_0$; the argument of J_n is $|\boldsymbol{r} - \boldsymbol{r}_0|$;

$$\Omega_{0n} = (\boldsymbol{\Omega}_0\cdot\boldsymbol{n}), \quad \Omega_{0\tau} = (\boldsymbol{\Omega}_0\cdot\boldsymbol{\tau}).$$

The last expression in (10) contains the particle surface temperature disturbance τ_s, which may be written here as $\tau_s(\theta_0) = \tau_{s0}\varphi(\theta_0)$, where τ_{s0} is no longer determined by angle θ_0. It follows from the linearity of the theory that the distribution function disturbance and hence any macroparameter Γ including gas–surface interaction parameters from (5) may be written in the form

$$\Gamma = \Gamma^U u_\infty + \Gamma^T \tau_{s0}. \tag{11}$$

Substituting (11) into (8)–(10) and extracting terms proportional to u_∞ and τ_{s0} splits the original equations (8)–(10) into two sets. The first describes the flow past a particle with uniform surface temperature equal to the free-stream gas flow temperature T_∞, and the second one describes the gas flow due to the surface temperature non-uniformity. Thus the original problem can be split. First the force $\boldsymbol{F}_U$ acting on the particle with the uniform surface temperature T_∞ in the moving gas should be determined taking into account the surrounding-gas thermal-polarization effect. The second problem is to estimate the radiometric force $\boldsymbol{F}_T$ acting on the non-uniformly heated particle is a motionless ($\boldsymbol{u}_\infty = 0$) gas.

4. Method of solution

The values of the radiometric force $\boldsymbol{F}_\mathrm{T}$ is dependent upon the $\varphi(\theta_0)$ function form. Let us expand it using the Legendre polynomials

$$\varphi(\theta_0) = \sum_{l=0}^{\infty} b_e P_e(\cos\theta_0). \tag{12}$$

Here it is convenient to choose τ_{s0} so that $b_1 = 1$. Then the macroparameters' angular dependence in the second problem (the quantities with the T superscripts) is also given in an expansion form analogous to (12). Owing to the Legendre polynomials' orthogonality, $\boldsymbol{F}_T$ is fully determined by the term in (12) with $l = 1$, for in the force definition (7) $V_z = V_r\cos\theta_0 - V_\theta\sin\theta_0$. Hence for both problems we may write

$$\begin{bmatrix} \nu^i \\ W_r^i \\ W_\theta^i \\ \tau^i \\ S_r^i \\ S_\theta^i \end{bmatrix} = \begin{bmatrix} \rho_1^i\cos\theta \\ \rho_2^i\cos\theta \\ \rho_3^i\sin\theta \\ \rho_4^i\cos\theta \\ \rho_5^i\cos\theta \\ \rho_6^i\sin\theta \end{bmatrix}, \quad \begin{bmatrix} a_0^i \\ a_{1n}^i \\ a_{1\tau}^i \\ a_2^i \end{bmatrix} = \begin{bmatrix} c_1^i\cos\theta_0 \\ c_2^i\cos\theta_0 \\ c_3^i\sin\theta_0 \\ c_4^i\cos\theta_0 \end{bmatrix}, \tag{13}$$

where $i = U, T$. Here $\rho_j^i(r)$ are the radial coordinate r functions, and c_j^i are the unknown constants independent of the θ_0 angle.

Both sets of integral-moment equations that the functions ρ_j^i satisfy are solved on the basis of the Bubnov–Galerkin method (Mikhlin 1970). The trial functions for the macroparameters are chosen according to the form of the hydrodynamic solution so as to ensure the quick convergence of the method at arbitrary Kn:

$$\begin{bmatrix} \tilde\rho_1^i \\ \tilde\rho_2^i \\ \tilde\rho_3^i \\ \tilde\rho_4^i \\ \tilde\rho_5^i \\ \tilde\rho_6^i \end{bmatrix} = \begin{bmatrix} c_5^i\dfrac{R^2}{r^2} \\ -\dfrac{R}{r}\left[\delta_{iU} - c_6^i\left(1-\dfrac{R^2}{r^2}\right)\right] \\ \dfrac{1}{2}\dfrac{R}{r}\left[\delta_{iU} - c_6^i\left(1+\dfrac{R^2}{r^2}\right)\right] \\ c_7^i\dfrac{R^2}{r^2} \\ c_8^i\dfrac{R^3}{r^3} \\ \tfrac{1}{2}c_8^i\dfrac{R^3}{r^3} \end{bmatrix}, \tag{14}$$

$$i = U, T; \quad \delta_{iU} = \begin{cases} 1, & i = U \\ 0, & i = T \end{cases}.$$

The unknown constants c_j^i are determined by substituting (13), (14) into the set of integral-moment equations (8)–(10); the expressions derived should be orthogonal with respect to each of the basic functions in (13), (14).

5. The particle surface-temperature determination

When the particle surface-temperature non-uniformity is due only to the particle thermal polarization in a gas flow and does not result from any inner heat source within the particle volume, the parameter τ_s in (2) is obtainable from a thermophysical problem solution. The temperature distribution over the particle volume is in this case described by the steady Laplace equation

$$\nabla^2 T_p(x, \theta_0) = 0. \tag{15}$$

The requirement of temperature limitation at the particle centre $T_p(x=0) < \infty$ and radial heat flow continuity at any point on the particle surface, where λ_p is the particle heat conductivity coefficient, are taken as the boundary conditions to (15):

$$-\lambda_p \frac{\partial T_p}{\partial x}\bigg|_{x=R_0} = P_\infty \left(\frac{2kT_\infty}{m}\right)^{\frac{1}{2}} \pi^{-\frac{3}{2}} \int c_r(c^2 - \tfrac{5}{2})\, h(R_0, \boldsymbol{c}) \exp(-c^2)\, \mathrm{d}\boldsymbol{c}. \tag{16}$$

Analysis of the boundary problem (15), (16) shows that the $\varphi(\theta_0)$ function in (12) in this case equals $\cos\theta_0$, i.e. the expressions (13) for $i = U$ provide the exact angular macroparameter dependence in gas.

The temperature drop between the ends of the particle diameter parallel to $\boldsymbol{U}_\infty$, with regard to the laws of conservation of mass and energy (6), is determined by

$$\Delta T = 2\tau_{s0} T_\infty = 2T_\infty u_\infty \alpha_E c_4^U / [\alpha_E(1 - c_4^T) + \tfrac{15}{4}(1 - \alpha_E) \Lambda Kn], \tag{17}$$

where $\Lambda = \lambda_p/\lambda_g$; λ_g is the heat conductivity of gas.

6. Discussion of results

In limiting cases of a viscous slip-flow ($Kn \ll 1$) and free-molecular ($Kn \rightarrow \infty$) regimes the analytical expressions for $\boldsymbol{F}_U$, $\boldsymbol{F}_T$ and ΔT can be obtained by the asymptotic expansion of the Galerkin coefficients, which are not given here because of their cumbersome nature. Numerical calculations were carried out at intermediate Kn and various α_E, α_n, α_τ, and Λ values. The computational error is within 1 % at all values of the variable parameter. The maximum error of the accepted approximation for the Bubnov–Galerkin method (13), (14) is estimated to be 3 %. The $\boldsymbol{F}_U$, $\boldsymbol{F}_T$ and ΔT dependence on the parameters α_E, α_n, α_τ and Λ is obtained here in an analytical form; their dependence on Kn for corresponding functions is given in tabular form.†

6.1. *Isothermal drag $\boldsymbol{F}_U$*

In a free-molecular regime ($Kn \rightarrow \infty$) the isothermal drag is

$$F_U^{\mathrm{FM}} = \tfrac{8}{3}\pi^{\frac{1}{2}} R_0^2 P_\infty U_\infty \left(\frac{m}{2kT_\infty}\right)^{\frac{1}{2}} \left\{2 + \alpha_\tau - \alpha_n \frac{32 - \pi(9 - \alpha_E)}{32 - \pi(1 - \alpha_n)(9 - \alpha_E)}\right\}. \tag{18}$$

† The tables for the dependence of $\boldsymbol{F}_U$, $\boldsymbol{F}_T$ and ΔT on Kn are not included in the present paper. Copies may be obtained by writing to the Editor of the *Journal of Fluid Mechanics* or to the authors.

In the case of complete momentum and energy accommodation ($\alpha_n = \alpha_\tau = \alpha_E = 1$) this expression becomes the well-known Epstein (1924) result.

In a viscous slip-flow regime ($Kn \ll 1$) the expression for F_U is

$$F_U = 6\pi\eta U_\infty R_0[1+aKn+bKn^2+O(Kn^3)], \tag{19a}$$

where a and b are numerical coefficients dependent on the accommodation coefficients. Within the computational accuracy a depends only on α_τ and is independent of α_n and α_E. In the case of complete accommodation $a = -1.137$; $b = 0.5650$. Analogous coefficients given by Sone & Aoki (1977) after suitable Kn transformations are $a = -1.147$ and $b = 0.6366$. This slight difference in a and b values results first from the fact that different kinetic equations were employed (the BGK equation by Sone & Aoki (1977) and Shakov equation in our work) and from different methods of solution. In the case $Kn \ll 1$ and $\alpha_\tau \rightarrow 0$ the numerical result confirmed the validity of the well-known Basset (1961) result

$$F_U = 4\pi\eta U_\infty R_0. \tag{19b}$$

At intermediate Kn the formula for the isothermal drag is written as

$$F_U^* = \frac{F_U^1+(1-\alpha_E)F_U^2}{F_U^3+(1-\alpha_E)F_U^4}, \tag{20}$$

where $$F_U^* = F_U/F_U^{\mathrm{FM}}(\alpha_n = \alpha_\tau = \alpha_E = 1),$$

$$F_U^n = \begin{cases} f_U^i & \text{at} \quad n \neq 3 \\ 1 & \text{at} \quad n = 3 \end{cases} + (1-\alpha_\tau)f_U^i + (1-\alpha_n)f_U^i + (1-\alpha_\tau)(1-\alpha_n)f_U^i,$$

where at $n = 1, i = 1, \ldots, 4$; at $n = 2, i = 5, \ldots, 8$; at $n = 3, i = 9, \ldots, 11$; and at $n = 4$, $i = 12, \ldots, 15$. Functions f_U^i depend only on Kn and are presented in tables A 1, A 2 (see footnote on p. 411).

At $\alpha_n = \alpha_\tau = \alpha_E = 1$, from (20) it follows that $F_U^* = f_U^1$; for F_U^* an approximate expression was obtained which agrees with the numerical calculations over the whole range of Kn within an error of 1%:

$$F_U^*(\alpha_n = \alpha_\tau = \alpha_E = 1) = \frac{Kn}{Kn+0.619}\left(1+\frac{0.310Kn}{Kn^2+1.152Kn+0.785}\right). \tag{21}$$

Comparison of the results obtained for cases of complete accommodation with other theories and the empirical Millikan (1923) formula is given in table 1. The two right-hand columns of table 1 provide the results of the numerical calculations of F_U^* (20) (Shakhov's 1974 equation taking into account the surrounding-gas thermal polarization) and the results of our previous paper (Beresnev *et al.* 1987, using the BGK (1954) equation without the surrounding-gas thermal polarization). The maximum discrepancy between these results does not exceed 0.2% over the whole range of Kn. This fact allows us to ignore the gas thermal-polarization effect on the isothermal drag. At the same time the difference between our present results (20) and the results of Law & Loyalka (1986) where the BGK equation was used and the thermal polarization of the gas was also taken into consideration, amounts to 6% at $Kn \approx 0.1$ We believe that such a discrepancy may be explained by the fact that the use of the BGK-model equation for computations of the gas thermal polarization near the particle is hardly justified, for it does not provide a correct description of the combined processes of heat and momentum transfer. The fact that the agreement

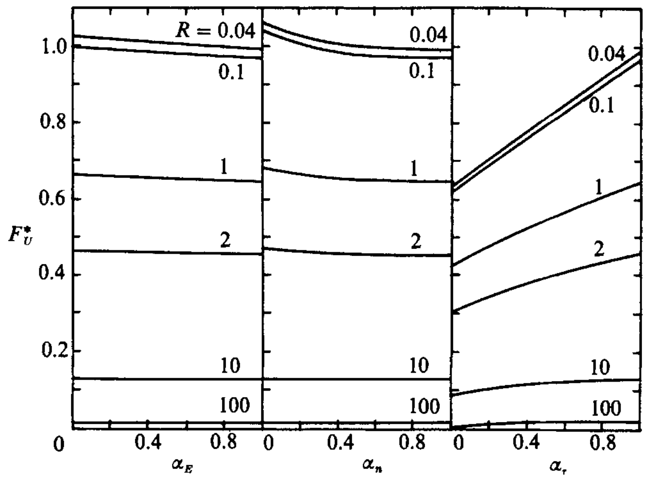

FIGURE 1. The accommodation dependence of the reduced isothermal drag F_U^* (equation (20)) at various $Kn = \frac{1}{2}\pi^{\frac{1}{2}}R^{-1}$.

R	Millikan (1923)	Cercignani *et al.* (1968)	Lea & Loyalka (1982)	Law & Loyalka (1986)	Sone & Aoki (1977)	Equation (21)	Equation (20)	Beresnev *et al.* (1987)
0.050	0.978	0.978	0.982	0.977	—	0.983	0.988	0.988
0.075	0.968	0.965	0.966	0.966	—	0.982	0.979	0.979
0.10	0.957	0.953	0.948	0.955	—	0.964	0.970	0.969
0.25	0.896	0.886	0.887	0.891	—	0.905	0.908	0.906
0.50	0.804	0.790	0.790	0.801	—	0.810	0.806	0.806
0.75	0.724	0.709	0.709	0.727	—	0.724	0.720	0.720
1.00	0.655	0.640	0.640	0.651	0.692	0.651	0.645	0.648
1.25	0.596	0.582	0.583	0.597	0.581	0.590	0.588	0.588
1.50	0.546	0.533	0.520	0.551	0.534	0.538	0.537	0.538
1.75	0.502	0.491	0.492	0.512	0.477	0.494	0.493	0.494
2.0	0.465	0.455	0.456	0.478	0.442	0.456	0.456	0.457
2.5	0.403	0.395	0.397	0.423	0.386	0.395	0.396	0.397
3.0	0.355	0.349	0.342	0.352	0.351	0.348	0.349	0.350
4.0	0.286	0.282	0.284	0.287	0.278	0.280	0.282	0.282
5.0	0.240	0.236	0.238	0.243	0.234	0.235	0.236	0.236
6.0	0.206	0.203	0.205	0.212	0.202	0.202	0.203	0.203
7.0	0.180	0.178	0.180	0.182	0.177	0.177	0.178	0.178
8.0	0.161	0.158	0.160	0.164	0.158	0.158	0.158	0.158
9.0	0.145	0.143	0.144	0.150	0.142	0.142	0.143	0.143
10.0	0.132	0.130	0.131	0.134	0.129	0.129	0.130	0.130

TABLE 1. Reduced to the free-molecular value isothermal drag F_U^* at $\alpha_n = \alpha_\tau = \alpha_E = 1$ as a function of inverse Knudsen number Kn, $R = \frac{1}{2}\pi^{\frac{1}{2}}Kn^{-1}$

between our present results (20) and Lea & Loyalka's (1982) results (the BGK equation without including the thermal polarization of the gas) at $Kn \approx 0.1$ is within 1 % also seems to support our reasoning.

In figure 1 the accommodation dependence of the reduced isothermal force F_U^* at various Kn is shown. The decreasing of the tangential momentum accommodation coefficient α_τ results in the decrease of tangential stresses on the particle surface, i.e.

in the reduction of the isothermal drag. The decrease in the normal momentum accommodation coefficient α_n leads to the growth of the angular non-uniformity of the normal stresses. This may be shown from the laws of conservation of the normal momentum on the particle surface (6). As a result the F_U^* force grows. The F_U^* dependence on α_n is a maximum in a free-molecular regime and is absent at $Kn \ll 1$.

The F_U^* dependence on the energy accommodation coefficient α_E is a result of the thermal polarization of gas flow past a particle with a uniform surface temperature. For an incomplete energy accommodation ($\alpha_E < 1$) the surrounding-gas thermal polarization increases. It leads to the redistribution of the normal stresses on the particle surface, which increases their angular non-uniformity. As a result the isothermal drag F_U^* increases. The F_U^* dependence on α_E is a maximum in a free-molecular regime, decreases in the intermediate and is absent in a viscous slip-flow regime.

6.2. *Radiometric force* F_T

In a free-molecular regime the radiometric force is due to the Knudsen mechanism: momentum transfer is on average greater for molecules reflected from the heated part of the particle surface than for those reflected from the cool part. The total uncompensated momentum gives a radiometric force in the direction of the 'cool' side of the particle. The expression for F_T as $Kn \to \infty$ is

$$F_T^{\mathrm{FM}} = \tfrac{32}{3}\pi R_0^2 P_\infty \frac{\alpha_E \alpha_n}{32 - \pi(1-\alpha_n)(9-\alpha_E)} \tau_{s0}. \tag{22}$$

At $Kn \ll 1$ the radiometric force is due to the thermal creep flow along the non-uniformly heated particle surface. The use of the Stokes–Fourier equation with the boundary conditions of thermal creep flow and a temperature jump on the particle surface gives the following expression for the radiometric force at $Kn \ll 1$:

$$F_T = 4\pi \frac{\eta^2}{\rho} \frac{K_{TS}}{(1+3C_m Kn)(1+2C_t Kn)} \tau_{s0}. \tag{23}$$

Here $K_{\tau s}$, C_m and C_t are respectively the thermal, isothermal creep flow and temperature jump coefficients; ρ is the mass gas density.

At intermediate Kn the reduced radiometric force value $F_T^* = F_T/F_T^{\mathrm{FM}}(\alpha_n = \alpha_\tau = \alpha_E = 1)$ was numerically calculated as a function of Kn and the accommodation coefficients. The F_T^* formula is of the following form:

$$F_T^* = \frac{\alpha_E F_T^1}{F_T^2 + (1-\alpha_E) F_T^3}, \tag{24}$$

where
$$F_T^1 = f_T^1 + (1-\alpha_T) f_T^2 + (1-\alpha_n) f_T^3 + (1-\alpha_\tau)(1-\alpha_n) f_T^4,$$
$$F_T^2 = F_U^3, \quad F_T^3 = F_U^4,$$

from (20). Functions f_T^i, $i = 1, \ldots, 4$ depend only on Kn and are given in table A 2 (see footnote on p. 411). At $Kn \ll 1$ for the case $\alpha_n = \alpha_\tau = \alpha_E = 1$ the analytical expression for F_T^* is derived by the asymptotic expansion of Galerkin coefficients from (24)

$$F_T^* = 8.594 Kn^2 - 16.341 Kn^3 + O(Kn^4). \tag{25}$$

It should be noted that at $Kn \ll 1$ the leading term $F_T^* \sim Kn^2$, while the leading term $F_U^* \sim Kn$.

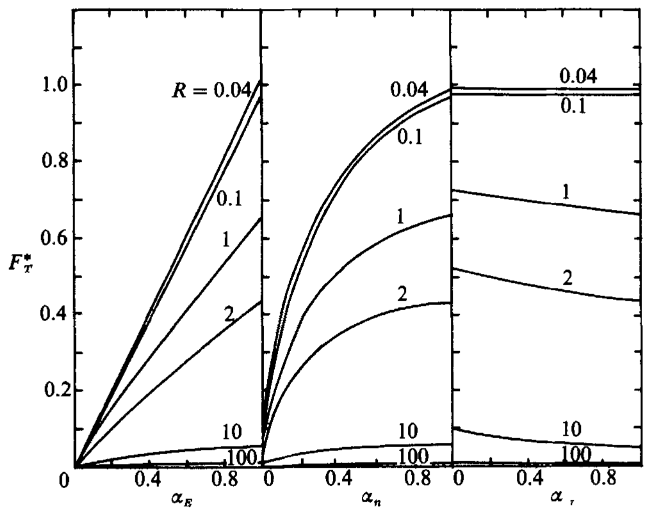

FIGURE 2. The accommodation dependence of the reduced radiometric force F_T^* (equation (24)) at various $Kn = \frac{1}{2}\pi^{\frac{1}{2}}R^{-1}$.

At $\alpha_\tau = \alpha_n = 1$ we obtain from (24) an expression for F_T^* that approximates the numerical calculation to within an error of 5% at all values of Kn and α_E:

$$F_T^* = \alpha_E \frac{Kn+0.129}{Kn+0.129\alpha_E} \frac{Kn^2}{Kn^2+0.116} \frac{Kn^2-0.222Kn+0.131}{Kn^2+0.015Kn+0.131}. \tag{26}$$

The accommodation dependence of F_T^* at various Kn is shown in figure 2. Owing to the incomplete molecular energy accommodation the gas is less susceptible to the particle surface-temperature non-uniformity. As a result the total uncompensated momentum transfer by gas molecules to the particle and the thermal creep flow along the non-uniformly heated surface decrease. The F_T^* value is maximum at $\alpha_E = 1$ and equals zero at $\alpha_E = 0$ at all Kn values.

Given the incomplete normal momentum accommodation the F_T^* value also decreases over the whole range of Kn.

The F_T^* dependence on α_τ is of some interest. With a decrease in α_τ the F_T^* value increases in the viscous slip-flow and intermediate regimes, and in the free-molecular regime it is independent of α_τ. The reason is that the normal and tangential stresses contribute to the radiometric force with opposite signs. The F_T^* value is mainly determined by the normal stresses, while the tangential stresses contribute to its decrease. The α_τ decrease results in a smaller tangential stress contribution, and hence to the radiometric force increase. At $Kn \approx 1$ the tangential stresses are already small, and as $Kn \to \infty$ they are totally absent. That is why at $Kn > 1$ the F_T^* dependence on α_τ is rather weak.

Schmitt (1961) has experimentally investigated the radiometric force affecting a macroscopic sphere over a wide range of gas pressures. During these experiments the temperature drop between the ends of the particle diameter was created by means of lighting one side of the particle, and it was kept constant at various gas pressures. The measurements were carried out in the range of $0.01 \lesssim Kn \lesssim 10$. Such experimental conditions ($\tau_{s0} =$ const at all Kn) allow a direct comparison of the theoretical data (24) with the experiment. In spite of the fact that in the experiment polyatomic gases were used (N_2, CO_2 etc.), the comparison has demonstrated a satisfactory qualitative agreement with the theory over the whole range of Kn values investigated. Given the complete momentum and energy accommodation the discrepancy does not exceed 15%. We are not aware of any other experimental results comparable with the theoretical data without the constant τ_{s0} value.

6.3. *The resulting force acting on the fixed particle*

The resulting force is determined by the relation

$$\boldsymbol{F} = \boldsymbol{F}^U u_\infty + \boldsymbol{F}^T \tau_{s0}. \tag{27}$$

The $\boldsymbol{F}^U$ direction is uniquely specified by the free-stream flow velocity vector $\boldsymbol{U}_\infty$, and the $\boldsymbol{F}^T$ direction is determined by the concrete temperature distribution on the particle surface. The radiometric force $\boldsymbol{F}^T$ is always directed from the hot to the cool side of the particle. Let us consider the case when the $\boldsymbol{F}^U$ and $\boldsymbol{F}^T$ vectors are oppositely directed. Then the criterion determining the magnitude and sign of the resulting force $\boldsymbol{F}$ can be obtained as

$$\tau_{s0}^+ = \frac{F^U}{F^T} u_\infty. \tag{28}$$

In the case $\tau_{s0} < \tau_{s0}^+$ the F value is positive and its direction coincides with the $\boldsymbol{U}_\infty$ direction. When $\tau_{s0} = \tau_{s0}^+$ F is zero, i.e. the isothermal drag is compensated by the oppositely directed radiometric force. And at $\tau_{s0} > \tau_{s0}^+$ the F force is negative, i.e. it is directed towards the gas flow.

6.4. *Particle thermal polarization in a gas flow*

In a free-molecular regime the temperature drop value ΔT between the ends of the particle diameter parallel to $\boldsymbol{U}_\infty$ is

$$\Delta T = \frac{16\pi^{\frac{1}{2}}\alpha_E \alpha_n T_\infty u_\infty}{32-\pi(1-\alpha_n)(9-\alpha_E)}\left[\frac{\lambda_p T_\infty}{R_0 P_\infty (2kT_\infty/\pi m)^{\frac{1}{2}}} + \alpha_E \frac{32-9\pi(1-\alpha_n)}{32-\pi(1-\alpha_n)(9-\alpha_E)}\right]^{-1}. \tag{29}$$

Through the asymptotic expansion of the Galerkin coefficients at $Kn \ll 1$, in the case $\alpha_n = \alpha_\tau = \alpha_E = 1$ we obtain the following expression for ΔT:

$$\Delta T = 2.031 T_\infty u_\infty Kn \frac{1+3.029Kn+7.115Kn^2}{2+\Lambda(1+4.931Kn+10.670Kn^2)}. \tag{30}$$

In the leading Kn term, (30) is in complete qualitative agreement with the analogous expression from Bakanov *et al.* (1979). The discrepancy between the numerical coefficients of (30) and the expression from Sone & Aoki (1977) does not exceed $\approx 2\%$ (2.031 and 2.076 respectively).

At intermediate Kn the formula for the reduced temperature drop has the form

$$\frac{\Delta T}{T_\infty u_\infty} = \frac{\alpha_E F_\Delta^1}{[\alpha_E + \frac{15}{4}(1-\alpha_E)\Lambda Kn] F_\Delta^2 + \alpha_E \Lambda F_\Delta^3}, \tag{31}$$

where

$$F_\Delta^m = \begin{cases} f_\Delta^i & \text{at} \quad m \neq 2 \\ 1 & \text{at} \quad m = 2 \end{cases} + (1-\alpha_\tau) f_\Delta^i + (1-\alpha_n) f_\Delta^i + (1-\alpha_\tau)(1-\alpha_n) f_\Delta^i,$$

where $m = 1,2,3$; $i = 1, \dots, 11$. At $m = 1, i = 1, \dots, 4$; at $m = 2, i = 5,6,7$; at $m = 3$, $i = 8, \dots, 11$. Functions f_Δ^i depend only on Kn and are given in table A 3 (see footnote on p. 411).

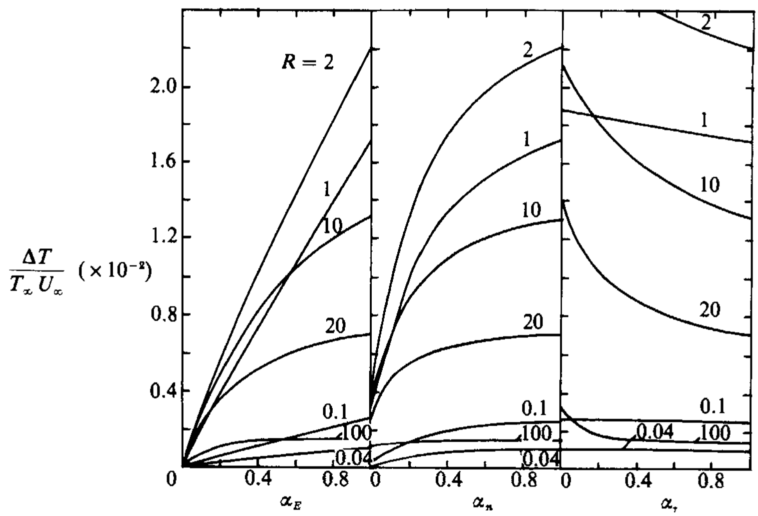

FIGURE 3. The accommodation dependence of the dimensionless temperature drop between the ends of the particle diameter $\Delta T/(T_\infty u_\infty)$ (equation (31)) at various $Kn = \frac{1}{2}\pi^{\frac{1}{2}}R^{-1}$ and $\Lambda = 10$.

At $\alpha_n = \alpha_\tau = 1$ we get from (31)

$$\frac{\Delta T}{T_\infty u_\infty} = \frac{\alpha_E f_\Delta^1}{\alpha_E + \frac{15}{4}(1-\alpha_E)\,\Lambda Kn + \alpha_E \Lambda f_\Delta^8}. \tag{32}$$

Choosing expressions for f_Δ^1 and f_Δ^8 in the form

$$\left.\begin{aligned} f_\Delta^1 &= \frac{18}{5\pi^{\frac{1}{2}}}\frac{Kn}{1+\dfrac{27}{\pi}Kn^2}\left[1+\frac{Kn}{3.028Kn^{2.14}+0.521}\right]^{-1}, \\ f_\Delta^8 &= \frac{2}{1+\frac{15}{2}Kn}\left[1+\frac{0.053Kn^{0.54}}{Kn^{1.62}+0.140}\right], \end{aligned}\right\} \tag{33}$$

we get agreement for the reduced temperature drop between (32) and the numerical calculation over the whole range of Kn within an error of 2% for all α_E and Λ.

The ΔT value is not a monotonic function of Kn. It grows as Kn increases in a viscous slip-flow regime, reaches its maximum value in the intermediate regime and then falls and stays constant (at fixed $\alpha_E, \alpha_n, \Lambda$) in the free-molecular regime. The position and magnitude of the ΔT maximum at intermediate Kn is determined by the heat conductivity parameter Λ and the accommodation coefficient values.

The dependence of the reduced temperature drop on the accommodation coefficients values $\alpha_E, \alpha_\nu, \alpha_\tau$ is given in figure 3. At $\alpha_E = 0$ over the whole range of Kn the ΔT value equals zero, as in this case the colliding molecules do not impart their energy to the particle. ΔT reaches its maximum value at $\alpha_E = 1$. It should be noted that the hydrodynamical theory (Bakanov *et al.* 1979, 1983) does not provide the correct limit transition $\Delta T \rightarrow 0$ as $\alpha_E \rightarrow 0$.

The α_n decrease also results in a ΔT decrease at all Kn; however at $\alpha_n = 0$ the ΔT value equals zero only as $Kn \rightarrow \infty$, and in other regimes it is finite. α_τ growth results in a ΔT decrease in the viscous slip-flow and intermediate regimes, and as $Kn \rightarrow \infty$ the ΔT value is independent of α_τ.

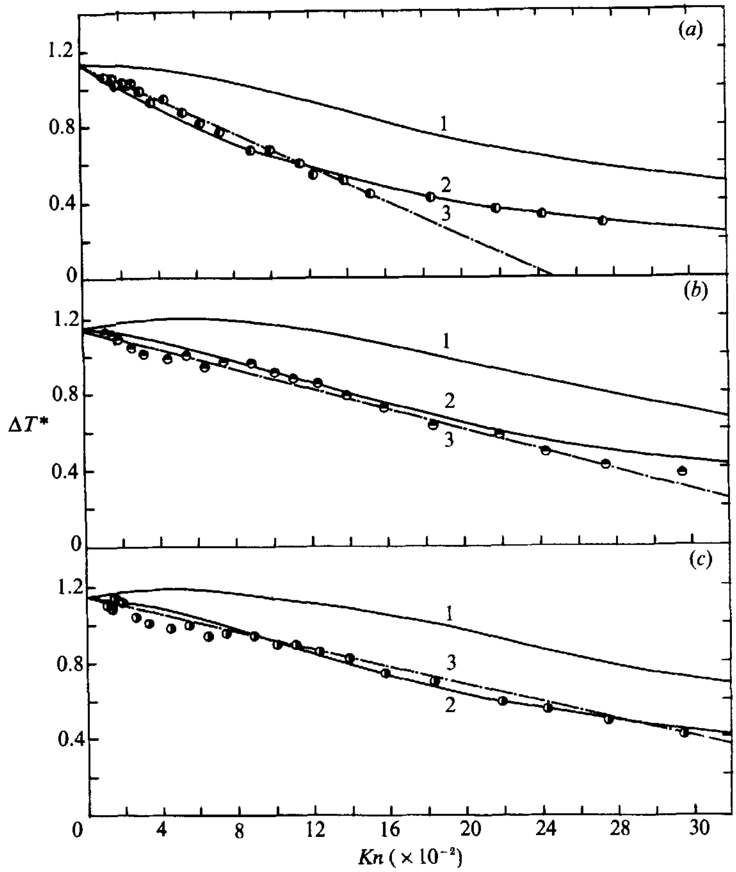

FIGURE 4. Comparison among the numerical calculation ΔT^* (equation (34)) and the Bakanov & Vysotsky (1980) Bakanov *et al.* (1983, 1986) experiment and the hydrodynamical theory of Bakanov *et al.* (1979, 1983). (*a*) He–Pyrex glass ($\Lambda = 7.5$); (*b*) He–vaseline-oil-covered organic glass ($\Lambda = 1.26$); (*c*) He–organic glass ($\Lambda = 1.26$). Curve 1, calculation using (34) at $\alpha_n = \alpha_\tau = \alpha_E = 1$; curve 2, calculation at $\alpha_n = 1$; $\alpha_\tau = 0.8$; $\alpha_E = 0.4$; curve 3, Bakanov *et al.* (1979, 1983) theory.

Comparisons of our results with experiment (Bakanov & Vysotsky 1980; Bakanov *et al.* 1983, 1986) and hydrodynamical theory (Bakanov *et al.* 1979, 1983) are presented in figures 4 and 5. For this purpose ΔT is scaled as

$$\Delta T^* = \Delta T(\Lambda+2)/(\pi^{\frac{1}{2}} Kn\, u_\infty T_\infty). \tag{34}$$

By substituting (31)–(33) into (34) we obtain the following expression for ΔT^* at $Kn \ll 1$:

$$\Delta T^* = 1.146\frac{(2+\Lambda)(1+3.029Kn+7.115Kn^2)}{2+\Lambda(1+4.931Kn+10.670Kn^2)}. \tag{35}$$

Equation (35) shows that in a viscous slip-flow regime the ΔT^* dependence on Kn is in general nonlinear. As $Kn \rightarrow 0$ $\Delta T^* = 1.146$ irrespective of the heat conductivity parameter Λ. At very small Kn values there is a small section of linear ΔT^* dependence on Kn; however, for low-conductivity particles ($\Lambda \lesssim 1$) the straight line inclination to the Kn axis tangent is positive, and for high-conductivity particles ($\Lambda \gtrsim 100$) it is negative. With Kn growth the ΔT^* dependence on Kn becomes non-

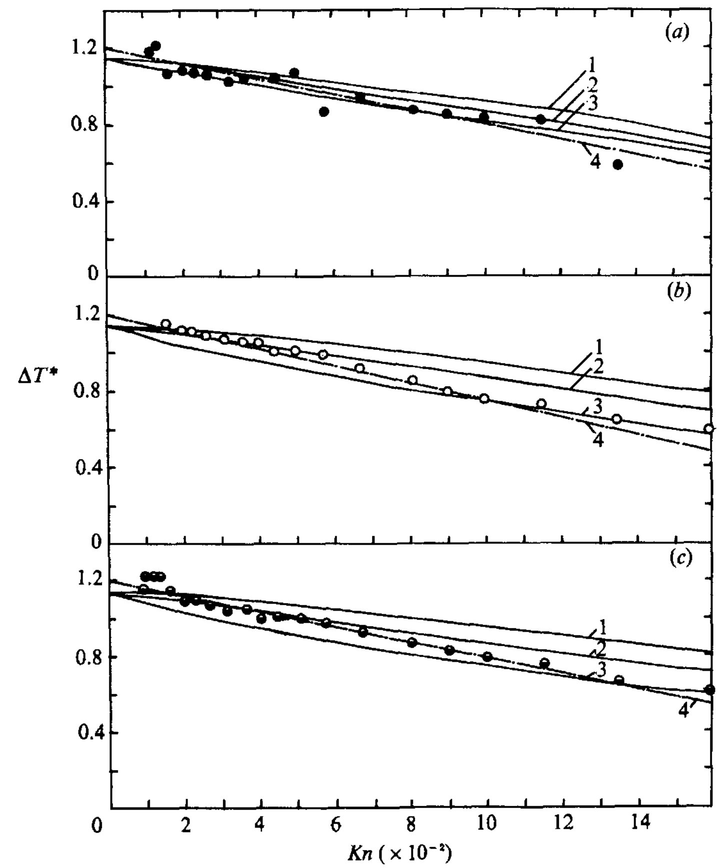

FIGURE 5. As figure 4, but for (*a*) Ar–Pyrex glass ($\Lambda = 62.5$); (*b*) Ar–organic glass with a thermally sprayed NaCl coating ($\Lambda = 10.5$); (*c*) Ar–vaseline-oil-covered organic glass ($\Lambda = 10.5$). Curve 1, calculation using (34) at $\alpha_n = \alpha_\tau = \alpha_E = 1$; curve 2, calculation at $\alpha_n = \alpha_\tau = 1$, $\alpha_E = 0.8$; curve 3, calculation at $\alpha_n = \alpha_\tau = 1$, $\alpha_E = 0.6$ for (*b, c*); at $\alpha_n = 1$; $\alpha_E = \alpha_\tau = 0.8$; for (*a*); curve 4, Bakanov *et al.* (1979, 1983) theory.

linear for both low- and high-conductivity particles. For low-conductivity particles (He–organic glass, $\Lambda = 1.26$; curve 1 in figure 4*b, c*) at $Kn \approx 0.04$–0.06 there is a weak maximum of ΔT^*. For moderate-conductivity particles (He–Pyrex glass, $\Lambda = 7.5$; curve 1 in figure 4*a*, Ar–organic glass, $\Lambda = 10.5$, curve 1 in figure 5*b, c*) at $Kn \approx 0.02$–0.04 there is a plateau for ΔT^*. With a further Kn increase the ΔT^* value monotonically decreases. For high-conductivity particles (Ar–Pyrex glass, $\Lambda = 62.5$, curve 1 in figure 5*a*) these dependencies are less apparent.

In the theoretical curves for the cases of incomplete momentum and energy accommodation these dependencies are also less apparent, but the ΔT^* dependence on Kn remains nonlinear. The experimental results seem on the whole to confirm this conclusion. The hydrodynamical thermal-polarization theory (Bakanov *et al.* 1979, 1983), being originally linear with respect to Kn, does not agree with the results of this kinetic analysis.

Comparison of the theory with experiment at real accommodation coefficient values ($\alpha_n = 1$; $\alpha_\tau = 0.8$–1.0; $\alpha_E = 0.4$–0.8) shows satisfactory agreement. However, the possibility of obtaining simultaneously all the three accommodation coefficients solely from the thermal-polarization experiments at $Kn \ll 1$ seems to be problematical.

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